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Number of iterations needed in Monte Carlo Simulation using reliability analysis for tunnel supports.

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ABSTRACT
There are many methods in geotechnical engineering which could take advantage of Monte Carlo Simulation to establish probability of failure, since closed form solutions are almost impossible to use in most cases. The problem that arises with using Monte Carlo Simulation is the number of iterations needed for a particular simulation.This article will show why it's important to calculate number of iterations needed for Monte Carlo Simulation used in reliability analysis for tunnel supports using convergence – confinement method. Number if iterations needed will be calculated with two methods. In the first method, the analyst has to accept a distribution function for the performance function. The other method suggested by this article is to calculate number of iterations based on the convergence of the factor the analyst is interested in the calculation.
Reliability analysis will be performed for the diversion tunnel in Rrëshen, Albania, by using both methods mentioned and results will be confronted.
***Keywords*:** convergence – confinement, iterations, Monte Carlo Simulation, probability of failure, reliability.

I. INTRODUCTION

Convergence - confinement method [1] is one of the most used methods to establish Factor of Safety for tunnel supports. Combining Monte Carlo Simulation [2] with this method, can be calculated probability of failure for the system of supports chosen. But, using Monte Carlo Simulation, is needed to accept the number of simulations needed. In an article form Ritter, Schoelles, Quigley, and Klein [3] is shown that nearly half (14 of 33 papers) presented in International Conference of Cognitive Modeling (2004), report no information at all about how many model runs were used.One way to calculate number of iterations in Monte Carlo Simulation, is to accept a distribution function for the random variable, usually is accepted as Normal distribution, and for this distribution is accepted confidence level and percentage error for calculating the mean value.This article will show two methods to calculate number of iterations needed in Monte Carlo Simulation, one that is used in other articles [4] and another one proposed in this paper.

II. NUMBER OF ITERATION NEEDED FOR MONTE CARLO SIMULATION.

If Monte Carlo Simulation is used, the analyst has to be sure of the results taken. For this, some way to calculate number of simulations needed has to be used, otherwise, the results may not be very reliable.

This article will show two methods to calculate number of iterations needed in Monte Carlo Simulation. Both methods are shown in the example of the reliability analysis performed for tunnelsupports used in the diversion tunnel in Rrëshen, Albania.2.1 Method 1. Number of iterations needed in Monte Carlo Simulation using Normal distribution for the performance function.

Normal Distribution is commonly used for statistical analysis in geotechnical engineering [2]. Method described in this paragraph uses Normal distribution as the distribution for the factor of safety for tunnel supports. Briefly this method is described here, but more extended information is available in other articles [4].

2.1.1 Confidence Level, Confidence Limits and Intervals.

From tables of the cumulative distribution function for a normally distributed random variable it may be seen that about 68% of the impacts should lie in the range $\pm\sigma$,95.5% will lie in the range $\pm 2\sigma$, and 99.7% in the range $\pm 3\sigma$. Fig. 1, gives the meaning of confidence limits and confidence levels. The percentage value is the confidence level, and the interval within which the value of x is expected to fall is the confidence limit.

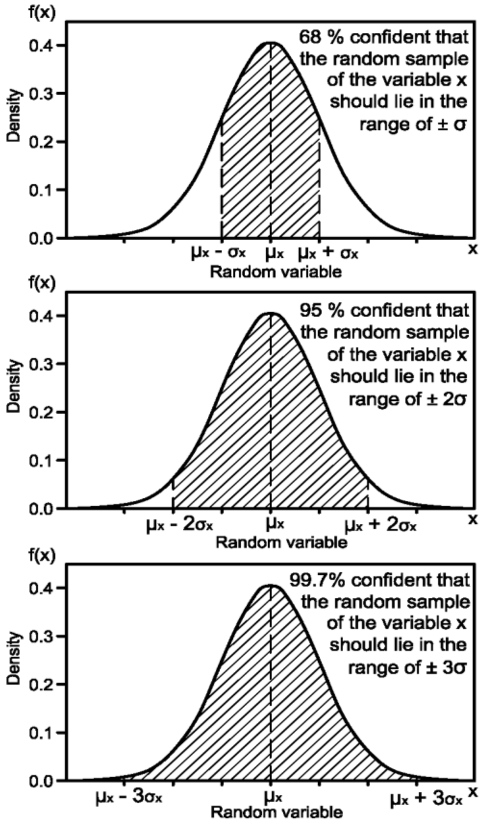

figure 1. Cumulative distribution function for normally distributed random variable. Confidence levels and confidence limits.

This range may be expressed in the form of an upper (U) and lower bound (L)
where:

$$U = \mu_x + z_c \sigma_x \quad (1)$$

$$L = \mu_x - z_c \sigma_x \quad (2)$$

Values of confidence coefficients zc for different confidence levels are given in Table 1.

Table 1. Values of zc for different confidence levels for a normally distributed random variable. [4].

Confidence level %	99.75	99	98	96	95.5
z_c	3	2.58	2.33	2.05	2
Confidence level %	95	90	80	68	50
z_c	1.96	1.645	1.28	1	0.6745

2.1.2 Confidence Intervals for the Mean and Error Bound.

The confidence interval, for a given confidence level, described above is an estimation of the variable x, a member of the population. It is also possible to generate confidence intervals for the mean of a population [4]. Equation 3 gives the expression to calculate the mean of a population.

$$(L, U) = \bar{x} \pm z_c \left(S_x / \sqrt{n} \right) \quad (3)$$

By considering the confidence interval to represent twice this maximum error can be written:

$$error_{max} = \frac{z_c S_x}{\sqrt{n}} \quad (4)$$

The percentage error of the mean becomes:

$$E = \frac{100 \times z_c S_x}{\bar{x}\sqrt{n}} \quad (5)$$

Solving for n:

$$n = \left[\frac{100 \times z_c S_x}{\bar{x} E} \right]^2 \quad (6)$$

n – is the number of samples. If used in Monte Carlo simulation, n is the number of iterations needed to perform, for a given error bound and confidence interval.

2.2. Another method to calculate number of iterations needed in Monte Carlo Simulation.

Accepting a distribution function for the factor of safety, may give not exact results, if the distribution accepted is very different that the one that has the performance function used in Monte Carlo Simulation. Accepting a distribution function can be avoided, as shown in paragraph 2.1, and number of iterations needed can be controlled by monitoring for what number of iterations, will the mean value, probability of failure and the standard deviation for the performance function converge.

III. PROBABILITY OF FAILURE FOR TUNNEL SUPPORT USING MONTE CARLO SIMULATION.

Monte Carlo Simulation will be used to calculate Probability of Failure for tunnel support. Monte Carlo Simulation uses random generated values to fill the series of values needed for calculation. In this study, random numbers will be generated with Normal Distribution. Random numbers will be generated using Excel software. From simulation, is calculated mean value and standard deviation for Factor of Safety. Probability of Failure can be calculated directly from cumulative distribution function which is constructed after the data calculated from Monte Carlo Simulation.

IV. RRËSHEN HYDROPOWER PLANT CASE STUDY. NUMBER OF ITERATIONS NEEDED FOR MONTE CARLO SIMULATION.

Hydropower plant of Rrëshen is being built in Fan River, in Rrëshen, Albania.Diversion tunnel has a length of 420m and a diameter of 9.1 m.Tunnel excavation was done by explosion, and face advance was from 0.5 to 2.5 m, depending on the type of rock.For every face advancement (in total 282 advances), has been made a face sketch and presented data for water, type of rock, joint number, joint alteration etc. From this data can be calculated RMR (Bieniawski), Q (Barton), GSI (Hoek& Brown).From boreholes near tunnel, have been taken rock samples and from them is calculated the Uniaxial Compression Strength (UCS). From rock description made from the geological engineer in site, is approximated the value of intact rock mi to be used in Hoek – Brown generalized failure criterion [5].For 282 tunnel face advances, have been calculated 282 values of GSI, from which are evaluated mean value and standard deviation. Taking into account that survey is conducted by two independent groups, dividing the tunnel in two parts, in this study will be taken into account only 173 values from the second group. From these 173 values, has been calculated mean and standard deviation. The same calculations are done for UCS. Using Marinos&Hoek table [6], is taken the value of mi, for gabbro and diabase is 10.Blast damage factor D is taken zero, because tunnel blast will be controlled and the rock can be assumed undisturbed.A sensitivity analysis was performed for UCS, GSI and mi. From sensitivity analysis, the parameters that influence most the calculations are UCS and GSI. Further calculation has been continued with these two parameters as uncertain.

Table 2: UCS and GSI values

	Mean Value	Standard Deviation
UCS (MPa)	46.62	10.6
GSI	26.24	7.09

Tunnel support will be constructed with steel ribs and shotcrete.Table 3 gives data for tunnel support.

Table 3. Data needed to construct Support Reaction Curve (SRC).

Support install distance from tunnel face	x = 0.5 m
Steel profile IPEA 330 with area	A = 0.005474 m²
Distance between steel profiles	i = 0.5 m
Shotcrete thickness	d = 0.15 m
Young modulus for concrete	$E_c = 2.5 * 10^7$kPa
Young modulus for steel	$E_s = 2.1 * 10^8$kPa
Steel yielding stress	$f_y = 5.4 * 10^5$kPa
Tunnel internal radius	R = 4.55 m

$$\frac{1}{K_c} = \frac{i \cdot R}{E_a \cdot A} + \frac{d}{E_c \cdot R} = 1.98037*10^{-6} \Rightarrow \quad Kc =$$
504956 kPa = 504.956 MPa

$$p_{\lim} = \frac{A \cdot \sigma_a}{i \cdot R} = 1299.323 \text{ kPa} = 1.299 \text{ MPa}$$

Radial deformation at the moment of support installation have been calculated using Vlachopoulos and Diederichs formulas [7].Ground Reaction Curve (GRC) has been constructed using Carranza – Torres and Fairhust method [8] which uses Hoek – Brown Generalized Failure Criterion [5].Calculation with mean values (deterministic method), has given the following results:
Ur (x = 0.5 m) = 9.78 mm and FS = 1.00 >= 1

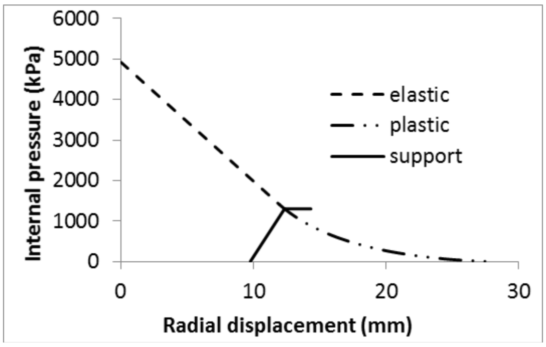

figure 2. Ground reaction curve (GRC) and Support Reaction Curve (SRC).

Using method described in paragraph 2.1, Monte Carlo simulation start with a low numbers of iterations, for example 5, after that they are done for 10, 50, 100, 200, 300 e so on, adding 100 for any simulation. In any of this simulations, Confidence Level and Error Bound is constant and the number of simulations needed can be calculated from equation (6). In all this simulation Confidence Level is 95% and Error Bound is E = 1%.

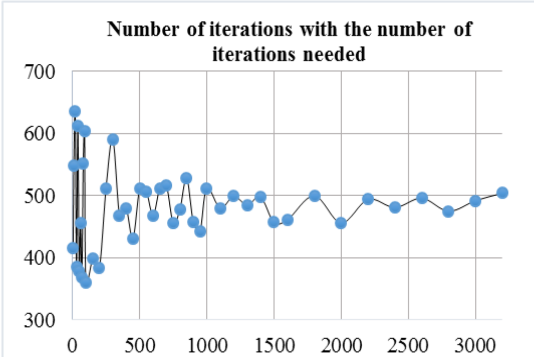

figure 3. Number of iterations with the number of iterations needed.

Calculation results are given in the graph of Fig. 3, where is seen that the number of iterations needed is around 500.Doing this simulations 10 times for 500 iterations, error bounds are as follows, 0.94, 0.93, 1.04, 1.02, 1.05, 1.01, 1.00, 0.98, 0.99, 1.00.From graph in Fig. 3, is shown that the number of iterations needed is stabilized between 500 to 1500 iterations, and for 100 iterations, is calculated 28% error from the calculated value of iterations needed.

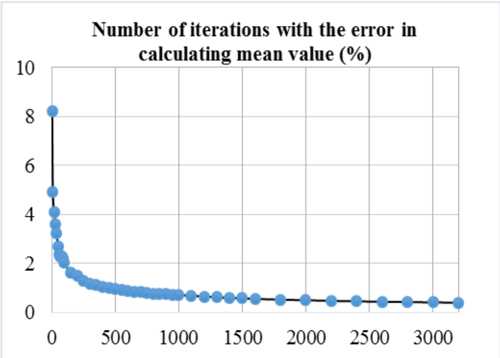

figure 4. Number of iterations with the error in calculating mean value.

From figure 4 is seen that increasing number of iterations, error in calculating mean value decreases. For 1 % error in calculating mean value, number of iterations is around 500.Method proposed by this paper, suggest to create graphs which will show for what number of simulations, the parameters the analystis interested to calculate will converge. Those parameters could be the mean value of factor of safety for tunnel supports (Fig. 5), standard deviation of factor of safety (Fig. 6), or probability of failure for the supports chosen for the tunnel (Fig. 7).

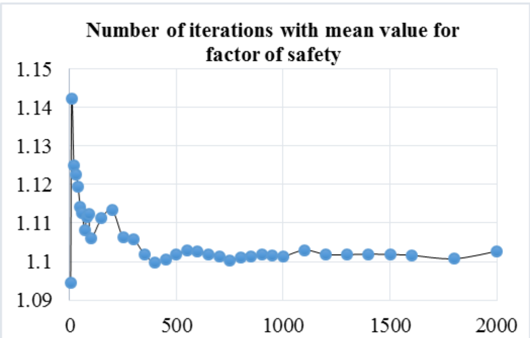

figure 5. Number of iterations with mean value for factor of safety.

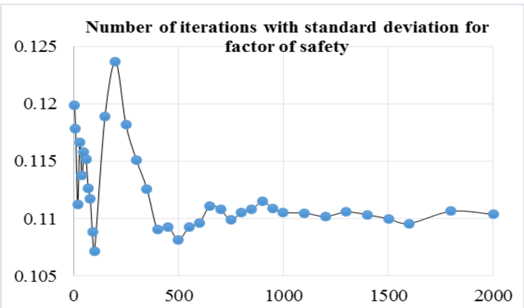

figure 6. Number of iterations with standard deviation for factor of safety.

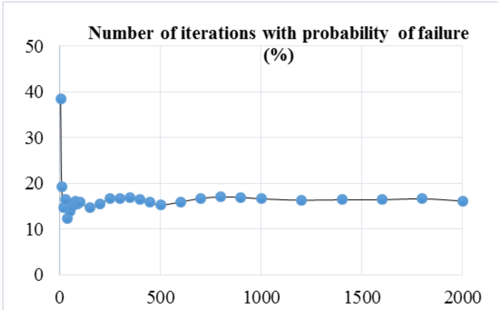

figure 7. Number of iterations with probability of failure for tunnel supports.

From Fig. 5, i see that the mean value for factor of safety converges after 500 iterations, which is the same value calculated with the method described in paragraph 2.1.Fig. 6 show that standard deviation for factor of safety, converges after 800 iterations.Probability of failure for tunnel supports is almost fixed after 200 iterations (Fig. 7).

V. CONCLUSIONS

Monte Carlo simulation should be associated with an analysis for number of iterations needed. If this analysis is not performed, error in calculating mean value can be very large. In our case error in calculating mean value, varies from 8 %, when 10 iterations are performed, to 0.4 %, for 3200 iterations (figure 4).

This article gives a procedure to establish number of iterations for Monte Carlo Simulation, by controlling the number of iterations based on the convergence of the factor the analystis interested. This factor could be the mean value of the safety factor for tunnel supports, probability of failure, or the standard deviation of safety factor.

As shown by this article, comparing the number of iterations needed for the convergence of the factor of safety (figure 5), is the same number (500 iterations) that is calculated using the method described in paragraph 2.1. Number of iterations needed for the probability of failure are 200 iterations and number of iterations needed for standard deviation of the factor of safety are 800 iterations.

Method to calculate number of iterations needed performing Monte Carlo Simulation, proposed in this article, does not need to accept a distribution function, so is a more global solution that the method described in paragraph 2.1.

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